International J.Math. Combin. Vol.1(2026), 01-26

New Mathematics over Combinatorial Structure – to Reductionism of Human

Linfan MAO

1. Chinese Academy of Mathematics and System Science, Beijing 100190, P.R. China
2. Academy of Mathematical Combinatorics & Applications (AMCA), Colorado, USA

E-mail: maolinfan@163.com

Abstract: Usually, one holds the reality of thing T by reductionism, namely subdividing T into elements by measuring their characters with solution of state equation to model its evolution. So, *can we understand the state of thing T by its one element or partial characters?* The answer is certainly Not because such an idea is only a hasty generalization unless there is only one element or all elements are in synchronizing, not being the case of self-organized system or biological system with number≥ 2 of elements in general, namely we lack a mathematics for adapting the recognition of reductionism. Notice that the subdividing of thing T naturally inherits a topological graph $\overrightarrow{G}$ in space, which is equivalently to transform thing T to a labelled graph $\overrightarrow{G}^L$, not the solution but the union of solutions of all state equations of elements, i.e., a Smarandache multuspace. Thus, we should establish a mathematics over $\overrightarrow{G}$ for characterizing the evolution of T, oblivious in the classic. Now, *could we regard $\overrightarrow{G}^L$ not only as a labelled graph but a mathematical element and establish such a mathematics that on $\overrightarrow{G}^L$?* The answer is certainly Yes, i.e., mathematical combinatorics following the CC conjecture, a combinatorial notion on classic mathematical extensions by reconstructed from or made by combinatorialization over topological graph $\overrightarrow{G}$. In this paper, we introduce the non-harmonious group corresponding to the reductionism and show how to establish a mathematical system over $\overrightarrow{G}$ with Smarandache multispace, including two types, i.e., $\overrightarrow{G}$ or its supergraph $\widehat{G}$ invariant for extending classic mathematics largely, including a few interesting results in functional analysis and Euler-Lagrange equation. All of these works constitute a mathematics to reductionism on combinatorial structure $\overrightarrow{G}$.

Key Words: Combinatorial notion, Smarandache multispace, CC conjecture, mathematical combinatorics, non-harmonious group, solution manifold, G-solution, continuity flow, Euler-Lagrange equation.

AMS(2010): 05C10, 05C21, 35A08, 46B25, 51D20, 51H20, 51P05.

§1. Introduction

As we all know, the developing of human depends on the recognition of thing T in nature, particularly by reductionism and the mathematics is such a formal system in logical consistency

[1]Reported at the *International Conference on New Age Mathematical Sciences*, Kolkata, India, December 11-13, 2025

[2]Received September 16, 2025, Accepted January 12, 2026

that can quantitatively model the evolution of thing T by its element so that the behavior of thing is consistent with that of its elements. Now, *could we really understand things by mathematics?* The answer is certainly Not by Godel's incompleteness theorem, namely there exist always statements in a formal system S that can neither be proved nor disproved so long as S contains the Peano's axioms of arithmetic. *Why is this case so happening?* Because all classic mathematics are incompleteness for adopting the Peano's axioms of arithmetic in default and ignores the inherited combinatorial structure $\overrightarrow{G}$ in reductionism.

1.1 Reductionism

The *reductionism* is a recognitive mechanism of human on things in the universe over years, i.e., subdividing thing T into the smallest elements of recognition.

Example 1.1 Matter M

For holding on the reality of a matter M, one subdivides M into molecule, atom, proton, neutron and finally, the elementary particles for recognition [32] such as the shown in Figure 1,

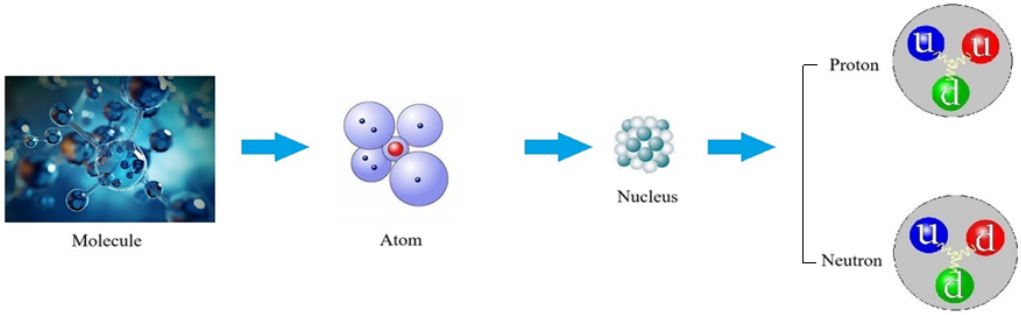

Figure 1.

i.e., subdividing

Matter M → Molecules → Atoms → Nucleus and Electronics → elementary particles.

Example 1.2 Living things L

There are two ways for recognition of living things, i.e., the microscopic and macroscopic recognition [39].

Microscopic Recognition. In this case, one subdivides L into cells and genes.

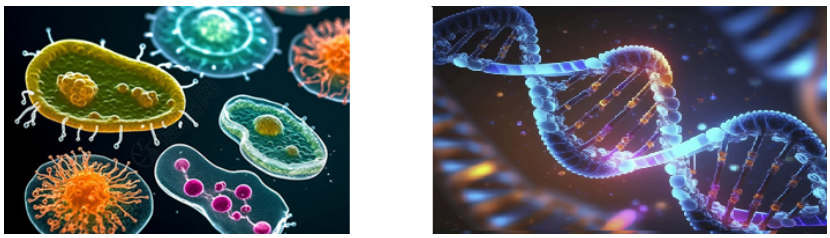

Figure 2.

i.e., subdividing

Living thing L → Biological Macromolecule → Cells and Genes.

Macroscopic Recognition. In this case, one views each one in its macroscopic state as an element. For example, the biological population.

Accordingly, the reductionism is equivalent matter T or living L to a complex network [2] underlying a 1-dimensional topological structure $\overrightarrow{G}$. Now, *could one hold on all behavior of things by such a complex network?* The answer is certainly Not. For example, the complex networks of male has 3.6×10^{13} cells and female has 2.8×10^{13} cells. If so, it will greatly increase the recognitive complexity, not the nature of thing but the artificial.

There is an assumption in reductionism without proof, namely the reality of thing T can be held on if the behavior of all elements are recognized. So, *could we conclude this assumption is right?* The answer is certainly Not because we can't assert T is the same of any one of its elements unless they are moving in synchronization such as those shown in Figure 3, where (a) fishes or (b) birds move from disorder to order.

(a) (b)

Figure 3.

However, elements in a self-organized system are usually not so, we can not assert it is the same of all elements in thing T because it inherits naturally a 1-dimensional topological structure $\overrightarrow{G}$ in reductionism.

Different from the subdividing on matter, living into smallest recognitive units or particles, the five elements [29], [40], i.e., the wood, fire, earth, metal and water of Chinese are five moving typing trends in nature. They are not particles or elements in periodic table of elements but five moving trends in nature. For examples, the character of fire is fierce, intense and rising, and the water is nourish, moisten and descend, i.e., the five elements provided the evolution mechanisms of things in the universe by the ancient Chinese.

Figure 4.

Furthermore, the five elements are not exist alone as particles but with interaction, i.e., generating and overcoming form 2 cycles as follows:

$$Wood \dashrightarrow Fire \dashrightarrow Earth \dashrightarrow Metal \dashrightarrow Water \dashrightarrow Wood \cdots, \text{ a cycle}$$

and

$$Wood \to Earth \to Water \to Fire \to Metal \to Wood \cdots, \text{ another cycle,}$$

where $\dashrightarrow$ denotes the generating action, $\to$ denotes the overcoming action, namely the whole evolution of thing T is under the mechanisms of 5 elements as shown in Figure 5.

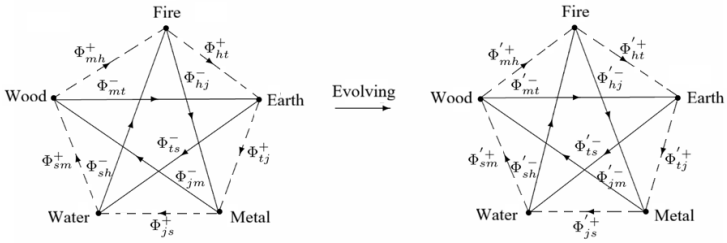

Figure 5.

1.2. Smarandache Multispace

Smarandache multispace is a recognitive notion on thing T from the local to the whole, against the limitation of human implied in the fable of six blind men with an elephant [28]. In this fable, each of the blind touched different part locally and claimed the different shape of an elephant, namely the 1st one claimed it is like a big smooth radish, the 2nd one claimed it is like a tube, the 3rd one claimed it is like a big fan, the 4th one claimed it is like a wall, the 5th one claimed it is like a big pillar and the 6th one claimed it is like a piece of rope. They stuck to their own views and then fell into an endless argument! So, *why are they arguing? why this case happens?* A sophist explained the reason to the blinds: *"Why you are thinking about the shape of elephant different is because each of you touches the different part of the elephant's body. Essentially, an elephant has all characters that you are talking about!"* Then, *what is an elephant shape in eyes of the sophist by locally recognition of the blind men?* The answer is the union of characters recognized by the 6 blind, i.e., the union of all the local to form a whole

Figure 6.

$$\begin{aligned}\text{An elephant} = &\{4\ big\ pillars\}\bigcup\{1\ gross\ rope\}\bigcup\{1\ tube\}\\ &\bigcup\{2\ big\ fans\}\bigcup\{1\ big\ wall\}\bigcup\{2\ big\ raddishes\},\end{aligned}$$

i.e., recognized by the sophist with the known character on that of the six parts of the six blinds. such as those shown in Figure 7

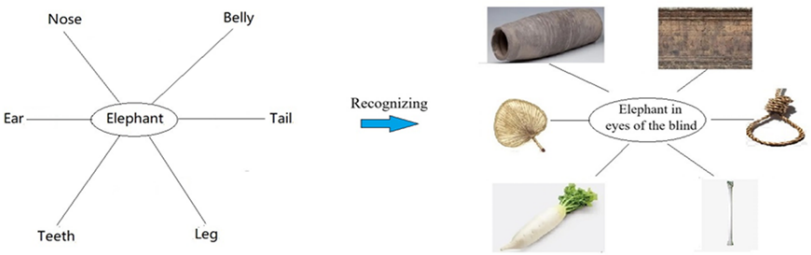

Figure 7.

with a tree structure

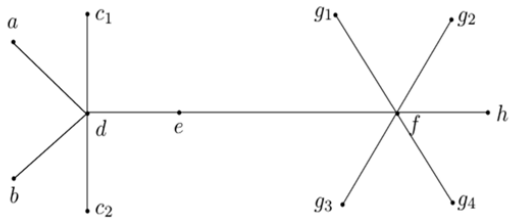

Figure 8

where $a = \{\text{teeth}\}$, $b = \{\text{nose}\}$, $c_1, c_2 = \{\text{ears}\}$, $d = \{\text{head}\}$, $e = \{\text{neck}\}$, $f = \{\text{belly}\}$, $g_1, g_2, g_3, g_4 = \{\text{legs}\}$, $h = \{\text{tail}\}$. Then, *what is a Smarandache multispace or multisystem?* Formally，a Smarandache multispace or multisystem is defined by

Definition 1.3([7],[33]) *Let $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ be m mathematical spaces or systems, different two by two, i.e., for any two spaces or systems $(\Sigma_i; \mathcal{R}_i)$ and $(\Sigma_j; \mathcal{R}_j)$, $\Sigma_i \neq \Sigma_j$ or $\Sigma_i = \Sigma_j$ but $\mathcal{R}_i \neq \mathcal{R}_j$. Then, a Smarandache multispace or Smarandache multisystem $\widetilde{\Sigma}$ is a union $\bigcup_{i=1}^{m} \Sigma_i$ with rules $\widetilde{\mathcal{R}} = \bigcup_{i=1}^{m} \mathcal{R}_i$ on $\widetilde{\Sigma}$, i.e., the union of rules $\mathcal{R}_i$ on Σ_i for integers $1 \leq i \leq m$, denoted by $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$.*

Such as the multisets that shown in Figure 9. In classical mathematics, algebraic rings, fields，topological groups, continuous groups are all cases of Smarandache multispace with $m = 2$.

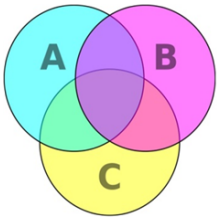

Figure 9

Now, *why Smarandache multispace or multisystem is important in recognition of human?* Because we are all blind in face of the unknown, as those of blind in fable of six blind men with an elephant. Generally, let the elements of thing T be $e_1, e_2, \cdots, e_m$ in reductionism and denote the mathematical reality of thing T by $T_{\mathcal{M}}$. So, one recognizes the mathematical reality of thing T by a union

$$T_{\mathcal{M}} = \bigcup_{i=1}^{m} \mathcal{R}(e_i)$$

of local recognitions of $e_1, e_2, \cdots, e_m$ by human, where $\mathcal{R}(e_i)$ is the recognition of e_i for integers $1 \leq i \leq m$. Then, *how to characterize $\mathcal{R}(e_i)$ by mathematics?* One applies mathematical system $(\Sigma_i, \mathcal{R}_i)$ for characterizing the evolving of element e_i. In this case, $T_{\mathcal{M}}$ is nothing else but a Smarandache multispace or multisystem.

Notice that any thing T inherits a topological structure of 1-dimension in its reductionism, which holds also for Smarandache multispace or multisystem, i.e., the intersection graph G.

Definition 1.4([7],[10]) *For an integer $m \geq 1$, let $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ be a Smarandache multispace or Smarandache systems consisting of m mathematical systems $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$. An inherited topological structure $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ of $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ is a labeled topological graph defined following:*

$$V\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{\Sigma_1, \Sigma_2, \cdots, \Sigma_m\},$$

$$E\left(G^L\left[\widetilde{\Sigma};\widetilde{\mathcal{R}}\right]\right) = \{(\Sigma_i,\Sigma_j)\,|\Sigma_i\bigcap\Sigma_j \neq \emptyset,\ 1 \leq i \neq j \leq m\}$$

with labeling

$$L:\ \Sigma_i \to L(\Sigma_i) = \Sigma_i,$$

$$L:\ (\Sigma_i,\Sigma_j) \to L(\Sigma_i,\Sigma_j) = \Sigma_i\bigcap\Sigma_j$$

for integers $1 \leq i \neq j \leq m$.

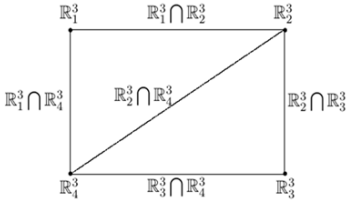

Figure 10

For example, a $K_4 - e$ system is shown in Figure 10.

1.3. CC Conjecture

CC conjecture is a philosophical notion for extending mathematics over combinatorial structure G. We know the subdivision of compact surface in surface topology following.

Theorem 1.5(Radó,1925, [30]) *Any compact surface S admits a triangulation.*

Theorem 1.5 implies that a 1-dimensional skeleton in a 2-cell partition of closed surface corresponds to a topological graph G and a 2-cell embedded of graph G on surface S is nothing else but a 2-cell partition of S which enables one getting the classification theorem following.

Theorem 1.6([30]]) *Any connected compact surface S is either homeomorphic to a sphere, or to a connected sum of tori, or to a connected sum of projective planes, i.e., its surface presentation $\mathcal{S}$ is elementary equivalent to one of the standard surface presentations following.*

(1) *the sphere* $S^2 = \langle a|aa^{-1}\rangle$;

(2) *the connected sum of p tori*

$$\underbrace{T^2\#T^2\#\cdots\#T^2}_{p} = \left\langle\ a_i,b_i,1\leq i\leq p\ |\ \prod_{i=1}^{p}a_ib_ia_i^{-1}b_i^{-1}\right\rangle;$$

(3) *the connected sum of q projective planes*

$$\underbrace{P^2\#P^2\cdots\#P^2}_{q} = \left\langle\ a_i,1\leq i\leq q\ |\ \prod_{i=1}^{q}a_i\right\rangle.$$

Theorem 1.7([30]) *Let S be a connected compact surface with a presentation $\mathcal{S}$. Then*

$$\chi(S) = \begin{cases} 2, & \text{if } \mathcal{S}\sim_{El} S^2,\\ 2-2p, & \text{if } \mathcal{S}\sim_{El} \underbrace{T^2\#T^2\#\cdots\#T^2}_{p},\\ 2-q, & \text{if } \mathcal{S}\sim_{El} \underbrace{P^2\#P^2\#\cdots\#P^2}_{q}. \end{cases}$$

Notice that the classification theorem of compact surface is just in one vertex, one face case, i.e. surface over a bouquet B_n as shown in Figure 11.

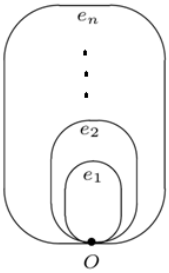

Figure 11

Generally, let G be a connected graph. Then, we have the results following.

Theorem 1.8 (Duke 1966, Stahl 1978, [4],[31]) *Let G be a connected graph and let $GR(G)$, $CR(G)$ be the respective genus range of G on orientable or non-orientable surfaces. Then, $GR(G)$ and $CR(G)$ both are unbroken interval of integers*

Theorem 1.8 has another form, namely any surface is 2-dimensional spaces $\mathbb{R}^2$ and generally, an n-dimensional manifold M is an n-dimensional space $\mathbb{R}^n$ over graph G. Generally, let $\overrightarrow{G}$ be a directed graph. It is this notion and Smarandache multispace inherited with a combinatorial structure that motivated me to make CC conjecture for extending mathematics widely in 2006.

Conjecture 1.9 (CC conjecture, Mao 2006, [6],[8]) *Any mathematical science can be reconstructed from or made by combinatorialization.*

Essentially, this CC conjecture is not a conjecture on mathematical problem but a philosophical notion for extending classic mathematics.

(1) It assumes that one can select finite combinatorial rulers with axioms to reconstruct or make a generalization for classical mathematics, and a mathematics is not complete if its combinatorialization be not completed.

(2) One can extend different mathematical systems over combinatorial structure $\overrightarrow{G}$ and find new mathematics corresponding to the theory of return to origin.reductionism, i.e.,

$$\textit{Mathematics over combinatorial structure } \overrightarrow{G} \Leftrightarrow \textit{ Reductionism}$$

Then, *could we really establish such mathematics over combinatorial structure $\overrightarrow{G}^L$?* The answer is certainly Yes, which needs to view $\overrightarrow{G}^L$ as an element of mathematics entirely by labeled graph $\overrightarrow{G}^L$. For examples, [12] on classification of n-dimensional manifold by listing graph, [9] and [10] on combinatorial manifold with differential, [11] on combinatorial fields and generally, mathematics on non-mathematics by combinatorics, all of them are motivated by CC conjecture.

The main purpose of this paper is to show how to establish such mathematics over combinatorial structure $\overrightarrow{G}$, i.e., mathematical combinatorics following the CC conjecture, including the mathematical model of non-harmonious group or equation's combinatorics corresponding to reductionism and how to establish a mathematical system over $\overrightarrow{G}$ with Smarandache multispace, particularly the algebra and calculus over an invariant $\overrightarrow{G}$ for extending algebra and calculus largely and generally, the continuity flow theory on a closed family $\mathscr{G}$ of graph union with Banach space $\mathscr{B}$ and G-isomorphic operators, i.e., $\overrightarrow{G}$ or its supergraph $\widehat{G}$ invariant for generalizing a few interesting results in functional analysis such as the closed graph theorem, the Hahn-Banach theorem and also the Euler-Lagrange equation. All of these works constitute a mathematics to reductionism on combinatorial structure $\overrightarrow{G}$.

For terminologies and notations not mentioned here, we follow references [1] for algebra, [2] for complex network, [3] for functional analysis, [30] for algebraic topology, [4],[5] and [31] for topological graphs, [32] for elementary particles, [7] and [33]-[35] for Smarandache multisystems and multispaces.

§2. Non-Harmonious Groups

Let T be a thing in the universe. *How do we characterize the physical state $\psi(t, \mathbf{x})$ of thing T by the reductionism, particularly a self-organized system?* Usually, a physical state of thing T is characterized by differential equation

$$\mathscr{F}\left(t, x_1, x_2, x_3, \psi_t, \psi_{x_1}, \psi_{x_2}, \psi_{x_3}, \cdots\right) \ = \ 0 \tag{2.1}$$

in physics ([15]), but *is it a solvable equation for self-organized system in mathematics?* The answer is certainly Not by the Godel's incompleteness theorem, i.e., it maybe characterized by a solvable system or a non-solvable system of solvable equations, namely, the solution of equation should be generalized. Then, *what is a non-harmonious group?* Let $\{e_1, e_2, \cdots, e_m\}$ be all elements of thing T in reductionism. Then, a non-harmonious group is formally defined in the following.

Definition 2.1([25],[28]) *A non-harmonious group $\mathcal{S}$ is such a mathematical system $\mathcal{S}$ consisting of elements $\mathbf{e}_i$, $1 \leq i \leq m, m \geq 2$ with interactions constrained on a system of equations*

$$(ES_m)\begin{cases} \mathscr{F}_{\mathbf{e}_1}(\mathbf{x}, \mathbf{y}) = 0, & (\mathbf{e}_1) \\ \mathscr{F}_{\mathbf{e}_2}(\mathbf{x}, \mathbf{y}) = 0, & (\mathbf{e}_2) \\ \cdots\cdots\cdots\cdots, & \cdots \\ \mathscr{F}_{\mathbf{e}_m}(\mathbf{x}, \mathbf{y}) = 0, & (\mathbf{e}_m) \end{cases} \tag{2.2}$$

at time t, where $\mathscr{F}_{\mathbf{e}_i}(\mathbf{x}^0, \mathbf{y}^0) = 0$ and $\mathscr{F}_{\mathbf{e}_i}$ satisfies the existence condition of implicit function theorem in a neighborhood U of point $(\mathbf{x}^0, \mathbf{y}^0)$ in Euclidean space $\mathbb{R}^n$ for integers $1 \leq i \leq m$.

Now, let $S_{\mathscr{F}_{\mathbf{e}_i}} \subset \mathbb{R}^n$ with $\mathscr{F}_{\mathbf{e}_i} : S_{\mathscr{F}_{\mathbf{e}_i}} \to 0$ be a solution manifold of the ith equation in system (ES_m). Then, *what is the condition of non-solvable or solvable of system (ES_m)?* The answer is

$$\bigcap_{i=1}^{m} S_{\mathscr{F}_{\mathbf{e}_i}} \ = \ \emptyset \quad \text{or} \quad \bigcap_{i=1}^{m} S_{\mathscr{F}_{\mathbf{e}_i}} \ \neq \ \emptyset, \tag{2.3}$$

i.e., the condition of non-solvable or solvable in geometrical space ([19]). So, *what is meaning of system (ES_m) has or has no solution? Is it a meaningless of thing T in the non-solvable case?* The answer is certainly not because the non-solvable of (ES_m) indicates only that there is no overlap state in elements $e_1, e_2, \cdots, e_m$ at time t, not implies the state of existing or not and solvable if the state of all elements have overlap, it is a serious matter characterizing the group behavior of $\{e_1, e_2, \cdots, e_m\}$. Then, *how to characterize the group behavior of system $\{e_1, e_2, \cdots, e_m\}$ or thing T?* The answer is nothing else but the Smarandache multispace $\bigcup_{i=1}^{m} S_{\mathscr{F}_{\mathbf{e}_i}}$, not the intersection or solution $\bigcap_{i=1}^{m} S_{\mathscr{F}_{\mathbf{e}_i}}$ of system (ES_m).

Notice that the group behavior $\bigcup_{i=1}^{m} S_{\mathscr{F}_{\mathbf{e}_i}}$ of system (ES_m) has combinatorial character, i.e., underlying a topological graph defined following.

Definition 2.2([9],[11]) *For any integer $m \geq 1$, the G-solution of system (ES_m) on non-harmonious group S is a labeled graph $G^L[ES_m]$ with vertex and edge sets defined by*

$$V\left(G^L[ES_m]\right) = \left\{S_{\mathscr{F}_{\mathbf{e}_i}},\ 1 \leq i \leq m\right\},$$

$$E\left(G^L[ES_m]\right) = \left\{\ (S_{\mathscr{F}_{\mathbf{e}_i}}, S_{\mathscr{F}_{\mathbf{e}_j}}) \mid if\ S_{\mathscr{F}_{\mathbf{e}_i}} \bigcap S_{\mathscr{F}_{\mathbf{e}_j}} \neq \emptyset\ for\ integers\ 1 \leq i, j \leq m\ \right\}$$

and labels on vertices and edges of G by

$$L:\ S_{\mathscr{F}_{\mathbf{e}_i}} \to S_{\mathscr{F}_{\mathbf{e}_i}}, \quad (S_{\mathscr{F}_{\mathbf{e}_i}}, S_{\mathscr{F}_{\mathbf{e}_j}}) \to S_{\mathscr{F}_{\mathbf{e}_i}} \bigcap S_{\mathscr{F}_{\mathbf{e}_j}}, \quad 1 \leq i \neq j \leq m.$$

Different from the usual, one can always conclude the existence of (ES_m) in this case.

Theorem 2.3([25],[28]) *For any integer $m \geq 1$, a G-solution $G^L[ES_m]$ of system (ES_m) on a non-harmonious group S is always existing.*

For example, let $(LDES_6^1)$ be a system of homogeneous differential equations $(\mathbf{e}_1) - (\mathbf{e}_6)$ as follows:

$$(LDES_6^1)\begin{cases} \ddot{x} - 3\dot{x} + 2x = 0 & (\mathbf{e}_1) \\ \ddot{x} - 5\dot{x} + 6x = 0 & (\mathbf{e}_2) \\ \ddot{x} - 7\dot{x} + 12x = 0 & (\mathbf{e}_3) \\ \ddot{x} - 9\dot{x} + 20x = 0 & (\mathbf{e}_4) \\ \ddot{x} - 11\dot{x} + 30x = 0 & (\mathbf{e}_5) \\ \ddot{x} - 7\dot{x} + 6x = 0 & (\mathbf{e}_6) \end{cases}$$

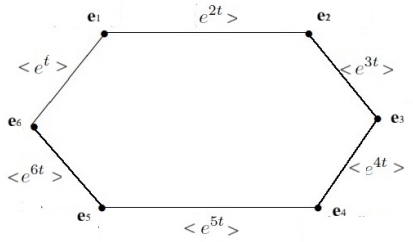

Figure 12

It is easily to know the solution space of $(1) - (6)$ are respectively

$$\left\langle e^t, e^{2t}\right\rangle, \left\langle e^{2t}, e^{3t}\right\rangle, \left\langle e^{3t}, e^{4t}\right\rangle, \left\langle e^{4t}, e^{5t}\right\rangle, \left\langle e^{5t}, e^{6t}\right\rangle, \left\langle e^{6t}, e^t\right\rangle$$

and

$$\begin{aligned} \left\langle e^t, e^{2t}\right\rangle \bigcap \left\langle e^{2t}, e^{3t}\right\rangle &= \left\langle e^{2t}\right\rangle, \quad \left\langle e^{2t}, e^{3t}\right\rangle \bigcap \left\langle e^{3t}, e^{4t}\right\rangle = \left\langle e^{3t}\right\rangle, \\ \left\langle e^{3t}, e^{4t}\right\rangle \bigcap \left\langle e^{4t}, e^{5t}\right\rangle &= \left\langle e^{4t}\right\rangle, \quad \left\langle e^{4t}, e^{5t}\right\rangle \bigcap \left\langle e^{5t}, e^{6t}\right\rangle = \left\langle e^{5t}\right\rangle, \\ \left\langle e^{5t}, e^{6t}\right\rangle \bigcap \left\langle e^{6t}, e^t\right\rangle &= \left\langle e^{6t}\right\rangle, \quad \left\langle e^{6t}, e^t\right\rangle \bigcap \left\langle e^t, e^{2t}\right\rangle = \left\langle e^t\right\rangle. \end{aligned}$$

Thus, if we remove the origin point O, the G-solution of $(LDES_6^1)$ can be shown in Figure 12.

So, *what is the role of the existence of G-solution on non-harmonious groups?* Its significance is to determine the global stability of non-harmonious group [13]. Let $G^L(t)$ be a G-solution of differential system (ES_m) with initial values $G^L(t_0)$ and let $\omega : V\left(G^L[ES_m]\right) \to \mathbb{R}$ be a functional. A system (ES_m) is said to be *ω-stable* if there exists a number $\delta(\varepsilon)$ for any number $\varepsilon > 0$ such that

$$\left\|\omega\left(G^{L_1(t)-L_2(t)}\right)\right\| < \varepsilon \tag{2.4}$$

or furthermore, *asymptotically ω-stable* if

$$\lim_{t\to\infty}\left\|\omega\left(G^{L_1(t)-L_2(t)}\right)\right\| = 0 \tag{2.5}$$

if the initial values hold with $\|L_1(t_0)(v) - L_2(t_0)(v)\| < \delta(\varepsilon)$ for $\forall v \in V(\overrightarrow{G})$ ([18]). In this case, if there is a Liapunov ω-function $L(\omega(t)) : \mathscr{O} \to \mathbb{R}, n \geq 1$ on $\overrightarrow{G}$ with open $\mathscr{O} \subset \mathbb{R}^n$ such that $L(\omega(t)) \geq 0$ with equality hold only if $(x_1, x_2, \cdots, x_n) = (0, 0, \cdots, 0)$ and $\dot{L}(\omega(t)) < 0$ if $t \geq t_0$.

Denoted by $\mathbf{O}$ the zero G-solution of system (ES_m), i.e., all vertices and edges on $G^L[ES_m]$ are labeled by $\mathbf{0}$, we get a result on ω-stability of (ES_m) following.

Theorem 2.4([18]) *If there is a Liapunov ω-function $L(\omega(t)) : \mathscr{O} \to \mathbf{R}$ on $G^L[ES_m]$ of system (ES_m), then it is ω-stable, and furthermore, if $\dot{L}(\omega(t)) < 0$ for $G^L[ES_m] \neq \mathbf{O}$, then it is asymptotically ω-stable.*

Particularly, let

$$\omega \;=\; \sum_{v\in V(G)} X_v(t) \quad \text{or} \quad \prod_{v\in V(G)} X_v(t) \tag{2.6}$$

on a linear system of differential equations (ES_m) [13]. Then, we have the sum-stability and prod-stability of linear system of differential equations following.

Definition 2.5([13]) *Let the inherited graph $G^L[ES_m]$ of linear systems (ES_m) of differential equation with be G initial value $X_v(0), v \in V(G)$. Then, G-solution $G^L[ES_m]$ is called sum-stable or prod-stable if for all solutions $Y_v(t)$, $v \in V(G)$ of the linear differential equations of (ES_m) with $|Y_v(0) - X_v(0)| < \delta_v$ exists for all $t \geq 0$,*

$$\left|\sum_{v\in V(G)} Y_v(t) - \sum_{v\in V(G)} X_v(t)\right| < \varepsilon \quad \text{or} \quad \left|\prod_{v\in V(G)} Y_v(t) - \prod_{v\in V(G)} X_v(t)\right| < \varepsilon \tag{2.7}$$

or furthermore, it is called asymptotically sum-stable or asymptotically prod-stable if

$$\lim_{t\to 0}\left|\sum_{v\in V(G)} Y_v(t) - \sum_{v\in V(G)} X_v(t)\right| = 0 \quad \text{or} \quad \lim_{t\to 0}\left|\prod_{v\in V(G)} Y_v(t) - \prod_{v\in V(G)} X_v(t)\right| = 0. \tag{2.8}$$

We know the result on sum-stability and prod-stability of linear system of differential equations following.

Theorem 2.6([13]) *A zero $\mathbf{O}$-solution of system (ES_m) of linear homogenous differential equation is asymptotically sum-stable if and only if* $\mathrm{Re}\alpha_v < 0$ *for each $\overline{\beta}_v(t)e^{\alpha_v t} \in \mathscr{B}_v$ with vertex $v \in G^L$ and a zero $\mathbf{O}$-solution of systems (ES_m) of linear homogenous differential equation is asymptotically prod-stable if and only if*

$$\sum_{v\in V(G)} \mathrm{Re}\alpha_v \;<\; 0 \tag{2.9}$$

for each $\overline{\beta}_v(t)e^{\alpha_v t} \in \mathscr{B}_v$ *with vertex* $v \in G^L[ES_m]$.

Certainly, different functionals ω on different combinatorial structure G will lead to different ω-stabilities of (ES_m) and there are open problems following.

Problem 2.7 *Construct different functional for characterizing* ω*-stability of non-harmonious groups* (ES_m).

Problem 2.8 *For a specific non-harmonious groups* (ES_m)*, construct functional* ω *on typical subgraphs of* $G^L[ES_m]$ *such as* P_2, P_3, C_3, K_4 *of small order.*

Meanwhile, characterizing the non-solvable system of equations of non-harmonious groups (ES_m) by combinatorics is a meaningful thing for holding on the reality of things T. For example, the non-solvable system of ordinary differential equations and the first order partial differential equations are respectively characterized in [13] and [18].

§3. Mathematics over an Invariant Graph

Let $\{e_1, e_2, \cdots, e_m\}$ be elements of thing T in reductionism and the evolving of elements $e_i, 1 \leq i \leq m$ are respectively characterized by mathematical systems $(\Sigma_1, \mathcal{R}_1), (\Sigma_2, \mathcal{R}_2), \cdots, (\Sigma_m, \mathcal{R}_m)$, namely, the pair $(\bigcup\limits_{i=1}^{m} \Sigma_i; \bigcup\limits_{i=1}^{m} \mathcal{R}_i)$ is a Smarandache multispace inherited with a topological structure G^L by Definition 1.4. Generally, $(\bigcup\limits_{i=1}^{m} \Sigma_i; \bigcup\limits_{i=1}^{m} \mathcal{R}_i)$ is not a mathematical system unless a union of mathematical systems, characterized by algebraic operations in $\bigcup\limits_{i=1}^{m} \mathcal{R}_i$ with operation diagram of elements [7], [36]-[37].

Now, *could we establish new mathematics over topological graph* G^L*?* The answer is certainly Yes by combinatorics! Generally, let $V(G^L) = \{v_1, v_2, \cdots, v_m\}$. Define labeled graphs G^{L*}, $\mathbf{o}_G$ respectively by elements in $\bigcup\limits_{i=1}^{m} \Sigma_i$ and operations in $\bigcup\limits_{i=1}^{m} \mathcal{R}_i$ on G^L with

$$L^* : v_i \to \alpha_i \in \Sigma_i, \quad \mathbf{o}_G : v_i \to \circ_i \in \mathcal{R}_i, \quad 1 \leq i \leq m \tag{3.1}$$

and

$$G_1^{L^*} \mathbf{o}_G G_2^{L^*} \ = \ G^{L_1^* \mathbf{o}_G L_2^*}. \tag{3.2}$$

Particularly, if $\mathcal{R}_i = \mathcal{R}$ for integers $1 \leq i \leq m$, i.e., each mathematical system $(\Sigma_i, \mathcal{R}_i)$ is with the same operation set $\mathcal{R}$ then the eq.(3.2) is simplified to

$$G_1^{L^*} \mathbf{o}_G G_2^{L^*} \ = \ G^{L_1^* \circ L_2^*}, \quad \circ \in \mathcal{R}. \tag{3.3}$$

Denoted by $\mathscr{G}^L$, $\mathbf{o}_{\mathscr{G}}$ all labeled graphs G^{L*} and all operations $\mathbf{o}_G$, respectively. Then, a Smarandache multispace $(\bigcup\limits_{i=1}^{m} \Sigma_i; \bigcup\limits_{i=1}^{m} \mathcal{R}_i)$ is convert to a system $(\mathscr{G}^L, \mathbf{o}_{\mathscr{G}})$ over combinatorial structure G^L. In this case, if we view G^{L^*} as a mathematical element in whole, we can establish new mathematics, particularly the algebra and calculus over G^L.

3.1. Algebra over an Invariant G

Let $(G_1;\circ_1),(G_2;\circ_2),\cdots,(G_m;\circ_m)$ be respectively groups with identities $1_{\circ_1},1_{\circ_2},\cdots,1_{\circ_m}$ ([1]). Then, a Smarandache multigroup is defined by

$$\left(\widetilde{G};\mathcal{O}\right) \ = \ \left(\bigcup_{i=1}^{m} G_i;\mathcal{O}\right) \tag{3.4}$$

with a combinatorial structure G^L, where $\mathcal{O}=\{\circ_i, 1\leq i\leq m\}$ and two multigroups $(\widetilde{G};\mathcal{O})$, $(\widetilde{G}';\mathcal{O}')$ are isomorphic if there is a $1-1$ isomorphism $\phi:\widetilde{G}\to\widetilde{G}'$ such that $\phi(G_i)=G_i'$ and $\phi(G_i\bigcap G_j)=\phi(G_i)\bigcap\phi(G_j)$, $1\leq i\neq j\leq m$.

Generally, a multigroup $(\widetilde{G};\mathcal{O})$ is not a group, even an algebraic system if it is not closed, i.e., $a\circ_i b\notin\widetilde{G}$ for some $\forall a,b\in\widetilde{G}$ and an integer $1\leq i\leq m$ in general. But it is a multigroup over combinatorial structure G^L. *Could we characterizes the multigroup by groups over combinatorial structure?* The answer is certainly Yes by group over combinatorial structure G^L.

Now, let $V(G^L)=\{v_1,v_2,\cdots,v_m\}$ be the vertex set of G^L. In this case, the labelling $L_1^*\mathbf{o}_G L_2^*$ in eq.(3.2)

$$G^{L_1^*}\mathbf{o}_G G^{L_2^*} \ = \ G^{L_1^*\mathbf{o}_G L_2^*}$$

is $L_1^*\mathbf{o}_G L_2^*: v_i\to L_1^*(v_i)\circ_i L_2^*(v_i)$ for integers $1\leq i\leq m$ is shown in Figure 13.

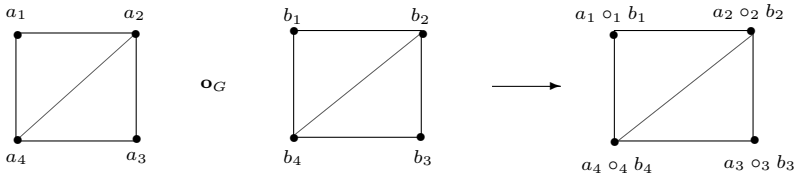

Figure 13

Clearly, the element $G^{L_I^*}$ with $L_I^*: v_i\to 1_{\circ_i}$ is the identity of $(G^L;\mathbf{o}_G)$ and for an element $G^{L^*}\in G^L$, $G^{L^{-*}}$ is its inverse, where $L^{-*}: L^*(v_i)\to L^{-*}(v_i)$ for integers $1\leq i\leq m$, and the associative

$$\left(G^{L_1^*}\mathbf{o}_G G^{L_2^*}\right)\mathbf{o}_G G^{L_3^*} \ = \ G^{L_1^*}\mathbf{o}_G\left(G^{L_2^*}\mathbf{o}_G G^{L_3^*}\right). \tag{3.5}$$

is implied by the associative law of group. Thus, $(G^L;\mathbf{o}_G)$ is a group by definition.

Theorem 3.1 *$(G^L;\mathbf{o}_G)$ is a group, i.e., a multigroup $(\widetilde{G};\mathcal{O})$ is a group over a combinatorial structure G^L of groups.*

For two isomorphic multigroups G^{L_1},G^{L_2}, if G^{L_1},G^{L_2} are isomorphic then there is a $1-1$ isomorphism $\phi:\widetilde{G}\to\widetilde{G}'$ such that $\phi(G_i)=G_i'$ and $\phi(G_i\bigcap G_j)=\phi(G_i)\bigcap\phi G_j$, $1\leq i\neq j\leq m$ by definition, namely the labeled graphs G^{L_1} and G^{L_2} are isomorphic with an isomorphism $\phi|_{v\in V(G)}$ of group. Thus, we have the following conclusion.

Theorem 3.2 *Two multigroups G^{L_1},G^{L_2} are isomorphic if and only if there is an isomorphism*

$\phi : G^{L_1} \to G^{L_2}$ *of graph such that*

$$\phi(L_1)(v) = L_2(\phi(v)), \quad v \in V(G) \tag{3.6}$$

and $\phi|_{v\in V(G)}$ *is an isomorphism of group.*

Problem 3.3 *Establish a multigroup* $(\widetilde{G}; \mathcal{O})$ *theory of commutative or non-commutative groups over a topological graph* G^L *such as* $G = K_m, P_m, C_m, \cdots$ *with small* $m = 2, 3, 4, \cdots$.

Particularly, if $\circ_1 = \circ_2 = \cdots = \circ_m = \circ$, namely all groups $(G_1; \circ_1), (G_2; \circ_2), \cdots, (G_m; \circ_m)$ are all subgroups of a group, i.e., the group is shown as in Figure 14, we get a corollary by Theorem 3.1 following.

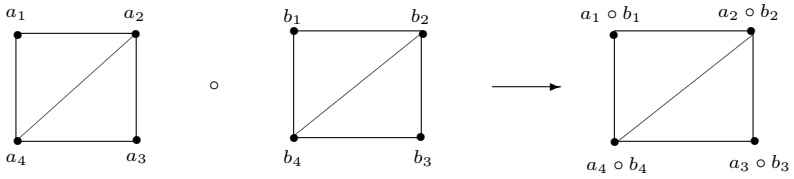

Figure 14

Corollary 3.4 *If* $\circ_1 = \circ_2 = \cdots = \circ_m = \circ$, *a multigroup* $(\widetilde{G}; \circ)$ *is a group, i.e., a group over a combinatorial structure* G *of subgroups.*

Problem 3.5 *Establish a multigroup* $(\widetilde{G}; \circ)$ *theory, determine all multigroups of a commutative or non-commutative group over a topological graph* G^L *such as* $G = K_m, P_m, C_m$ *with small* $m = 2, 3, 4, \cdots$.

Similarly, we can also establish multialgebra by rings, skew field and field. Let $(R_1; \{+_1, \cdot_1\})$, $(R_2; \{+_2, \cdot_m\}), \cdots, (R_m; \{+_2, \cdot_m\}))$ be respectively rings, skew fields or field with "+" identities $0_{+_1}, 0_{+_2}, \cdots, 0_{+_m}$ and "$\cdot$" identities $1_{\cdot_1}, 1_{\cdot_2}, \cdots, 1_{\cdot_m}$. Then, a Smarandache multiring, skew multifield or a multifeld is defined by

$$\left(\widetilde{R}; \mathcal{O}^2\right) \ = \ \left(\bigcup_{i=1}^{m} R_i; \mathcal{O}^2\right)$$

with a combinatorial structure G^L by Definition 1.4, where $\mathcal{O}^2 = \{+_G, \cdot_G\}$ and

$$+_G = \bigcup_{i=1}^{m} \{+_i\}, \ \cdot_G = \bigcup_{i=1}^{m} \{\cdot_i\}.$$

Two multirings, skew multifields or multifields $(\widetilde{R}; \mathcal{O}^2), (\widetilde{R}'; \mathcal{O}'^2)$ are isomorphic if there is a $1-1$ isomorphism $\phi : \widetilde{R} \to \widetilde{R}'^2$ such that $\phi(R_i) = R'_i$ and $\phi(R_i \bigcap R_j) = \phi(R_i) \bigcap \phi R_j$, $1 \le i \ne j \le m$. In this case, the operation (3.2) on G^{L^*} is

$$G^{L_1^*} \mathbf{o}_G G^{L_2^*} \ = \ G^{L_1^* \mathbf{o}_G^2 L_2^*}, \tag{3.7}$$

where the labelling $L_1^*\mathbf{o}_G^2L_2^* : v_i \to \{L_1^*(v_i) +_i L_2^*(v_i), L_1^*(v_i) \cdot_i L_2^*(v_i)\}$ for integers $1 \leq i \leq m$ shown in Figure 15.

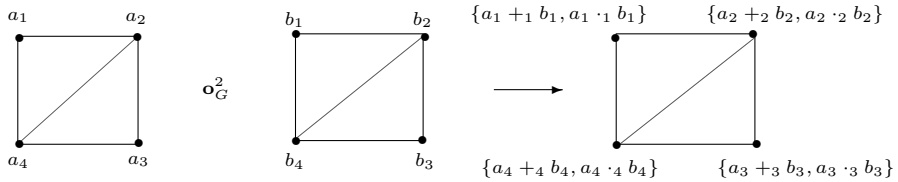

Figure 15

Clearly, the elements $G^{L_I^+}$ and $G^{L_I^\cdot}$ with $L_I^+ : v_i \to 1_{+_i}$, $L_I^\cdot : v_i \to 1_{\cdot_i}$ are respectively the identity of $(G^L; \mathbf{o}_G^2)$ of operations $+$ and $\cdot$, and for an element $G^{L^*} \in G^L$, G^{-L^*} and $G^{L^{-\cdot}}$ are respectively the inverses of G^{L^*} in $(R_i; +_i)$, $(R; \cdot_i)$. Furthermore, the associative

$$\left(G^{L_1^*}\mathbf{o} * 2_G G^{L_2^*}\right)\mathbf{o}_G^2 G^{L_3^*} \;=\; G^{L_1^*}\mathbf{o}_G^2\left(G^{L_2^*}\mathbf{o}_G^2 G^{L_3^*}\right), \tag{3.8}$$

and distributive

$$\begin{aligned} G^{L_1^*} \cdot_G (G^{L_2^*} +_G G^{L_3^*}) &= G^{L_1^*} \cdot_G G^{L_2^*} + G^{L_1^*} \cdot_G G^{L_3^*}, \\ (G^{L_2^*} +_G G^{L_3^*}) \cdot_G G^{L_1^*} &= G^{L_2^*} \cdot_G G^{L_1^*} +_G G^{L_3^*} \cdot_G G^{L_1^*} \end{aligned} \tag{3.9}$$

are respectively implied in the associative and distributive laws of ring, skew field or field by definition.

Theorem 3.6 *$(G^L; \mathbf{o}_G^2)$ is a ring, skew field or field, i.e., a multiring, skew multifield or multifield $(\widetilde{R}; \mathcal{O}^2)$ is a ring, skew field or field over a combinatorial structure G^L of rings, skew fields or fields.*

Similar to Theorem 3.2, we have the criterion for isomorphic multirings, skew multifields or multifields following.

Theorem 3.7 *Two multirings, skew multifields or multifields G^{L_1}, G^{L_2} are isomorphic if and only if there is an isomorphism $\phi : R^{L_1} \to R^{L_2}$ of graph such that*

$$\phi(L_1)(v) = L_2(\phi(v)), \quad v \in V(G) \tag{3.10}$$

and $\phi|_{v\in V(G)}$ is an isomorphism of ring, skew field or field.

Particularly, if $+_1 = +_2 = \cdots = +_m = +$ and $\cdot_1 = \cdot_2 = \cdots = \cdot_m = \cdot$, namely all rings, skew fields or fields $(R_1; \{+1, \cdot_1\}), (R_2; \{+2, \cdot_2\}), \cdots, (R_m; \{+m, \cdot_m\})$ are subrings, skew subfields or subfield of a ring, a skew field or a field, respectively.

Corollary 3.8 *If $+_1 = +_2 = \cdots = +_m = +$ and $\cdot_1 = \cdot_2 = \cdots = \cdot_m = \cdot$, a multiring, skew multifield or multifield $(\widetilde{G}; \circ)$ is a ring, skew field or field, i.e., a multiring, skew multifield or multifield $(\widetilde{R}; \mathcal{O}^2)$ is a ring, skew field or field over a combinatorial structure G of subrings,*

skew subfields or subfields.

Problem 3.9 *Establish a multiring, skew multifield and multifield* $(\widetilde{R};\mathbf{o}_G^2)$ *theory over a topological graph* G^L *such as* $G = K_m, P_m, C_m, \cdots$ *with small* $m = 2, 3, 4, \cdots$.

3.2. Calculus over an invariant G

Certainly, all elements of thing T in reductionism are not separated but interactive, i.e., one should characterize simultaneously the elements with interaction for holding on the evolving of T but the algebra over an invariant G^L can not take on this responsibility because it fails to characterize the interaction between elements. For this objective, a useful way is by G-flows on edges of G^L holding with conservative laws at each vertex of G^L, i.e., replacing the interaction by substance flow in microscopic. For example, the generating or overcoming interaction with wood inadequacy or excessiveness of Chinese 5 elements are shown in Figure 16.

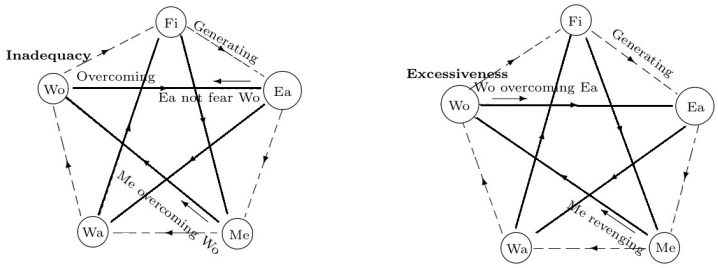

Figure 16

where Wo = Wood, Fi = Fire, Ea = Earth, Me = Metal and Wa = Water.

So, *what is a G-flow?* Generally, let $\mathscr{V}$ be a vector space over field $\mathscr{F}$. A *G-flow* is a labeled graph $\overrightarrow{G}^L$ ([16]) with a labeling mapping

$$L : v \to L(v) \in \mathscr{V},\ \forall v \in V(G) \ \text{ and } \ (v,u) \to L(v,u) \in \mathscr{V},\ \forall (v,u) \in E(G),$$

holding with $L(v,u) = -L(u,v)$, called the *vertex flow* and *edge flow* such as those shown in Figure 17,

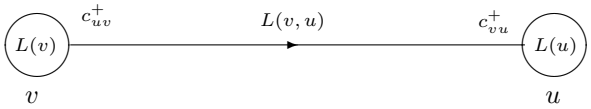

Figure 17

where the action of end-scalars $c^+_{vu}, c^+_{uv} \in \mathscr{F}$ on flow $L(v,u)$ is

$$c^+_{vu} : L(v,u) \to c^+_{vu}L(v,u), \quad c^+_{uv} : L(u,v) \to c^+_{uv}L^{A^+_{uv}}(u,v)$$

and meanwhile, for any vertex $v \in V(G^L)$ holding with conservative law

$$\sum_{u\in N^-_G(v)} c^+_{uv}L\,(u,v) - \sum_{u\in N^+_G(v)} c^+_{vu}L\,(v,u) \ = \ \mathbf{c}(v), \tag{3.11}$$

where $N_G^-(v)$ and $N_G^+(v)$ are respectively the in-neighborhood and out-neighborhood of vertex $v \in V(G)$, i.e., adjacent vertices $N_G^-(v) \subset N_G(v)$ flow into v and adjacent vertices $N_G^+(v) \subset N_G(v)$ flow out v with $N_G^-(v) \cup N_G^+(v) = N_G(v)$, and $\mathbf{c}(v)$ is the conservative vector at vertex v.

So, *are there really G-flows in the universe?* The answer is certainly Yes! For example, the 12 meridians with 671 acupoints on human body in Chinese medicine shown in Figure 18, which is nothing else but a G-flow of 671 vertices ([29]). Generally, we establish calculus on G-flows for vector space $\mathscr{V}$ over a field $\mathscr{F}$ to measure the microscopic changing of G-flow ([25],[28]).

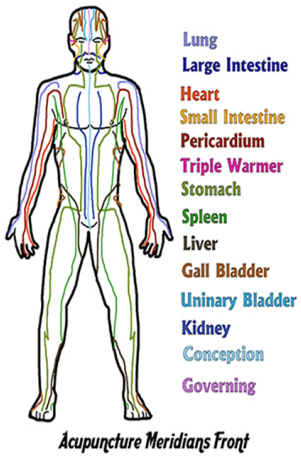

Figure 18

(1) Operations

Define the addition "+" and multiplication "·" by

$$\overrightarrow{G}^L + \overrightarrow{G}^{L'} = \overrightarrow{G}^{L+L'}, \quad \overrightarrow{G}^L \cdot \overrightarrow{G}^{L'} = \overrightarrow{G}^{L\cdot L'}, \tag{3.12}$$

where the mappings $L + L'$ and $L \cdot L'$ are respectively defined by

$$L + L'(v,u) = L(v,u) + L'(v,u), \quad L' \cdot L(v,u) = L'(v,u)L(v,u) \tag{3.13}$$

for any edge $(v,u) \in E(G)$.

(2) Distance

$$\rho\left(\overrightarrow{G}^L, \overrightarrow{G}^{L'}\right) = \sqrt{\sum_{(v,u)\in E(G)} (L(v,u) - L'(v,u))^2} = \rho\left(\mathbf{O}, \overrightarrow{G}^{L-L'}\right), \tag{3.14}$$

and particularly, the *norm* $\left|\overrightarrow{G}^L\right|$ of G-flow $\overrightarrow{G}^L$ in $\overrightarrow{G}^{L'} = \mathbf{O}$, i.e.,

$$\left|\overrightarrow{G}^L\right| = \rho\left(\mathbf{O}, \overrightarrow{G}^L\right) = \sqrt{\sum_{(v,u)\in E(G)} L^2(v,u)}. \tag{3.15}$$

(3) Continuous Mapping

A continuous mapping on G-flow is designed to characterize the changing caused by a slight changing of the G-flow. For two continuous evolving $\overrightarrow{G}^L[\mathbf{x}], \overrightarrow{G}^{L'}[\mathbf{x}]$ of G-flow, let f be a mapping of $\overrightarrow{G}^L[\mathbf{x}]$. If for any positive number ε there is a real number δ such that

$$\rho\left(\overrightarrow{G}^L[\mathbf{x}], \overrightarrow{G}^{L'}[\mathbf{x}_0]\right) < \delta \quad \Rightarrow \quad \rho\left(f\left(\overrightarrow{G}^L[\mathbf{x}]\right), \overrightarrow{G}^{L'}[\mathbf{x}_0]\right) < \varepsilon, \tag{3.16}$$

then f is said to be *continuous* at $\overrightarrow{G}^{L'}[\mathbf{x}_0]$, denoted by

$$\lim_{\overrightarrow{G}^L \to \overrightarrow{G}^{L'}} f\left(\overrightarrow{G}^L[\mathbf{x}]\right) = \overrightarrow{G}^{L'}[\mathbf{x}_0]. \tag{3.17}$$

(4) Differential

Let $\overrightarrow{G}^L[t]$ be a continuous evolving of G-flow on t with a continuous mapping f and $f(\overrightarrow{G}^L[t+\Delta t]) \to f\left(\overrightarrow{G}^L[t]\right)$ if $\Delta t \to 0$. In this case, if the limitation

$$\lim_{\Delta t\to 0} \frac{f\left(\overrightarrow{G}^L[t+\Delta t]\right) - f\left(G^L[t]\right)}{\overrightarrow{G}^{\Delta t}} \;=\; \overrightarrow{G}^{\lim\limits_{\Delta t\to 0}\frac{f(L)[t+\Delta t]-f(L)[t]}{\Delta t}}$$

exists, it is said that f is *differential* at G-flow $\overrightarrow{G}^L[t]$, denoted by

$$\frac{df}{dt} \;=\; \lim_{\Delta t\to 0} \frac{f\left(\overrightarrow{G}^L[t+\Delta t]\right) - f\left(\overrightarrow{G}^L[t]\right)}{\overrightarrow{G}^{\Delta t}}, \tag{3.18}$$

where $\Delta : (v,u) \in E(G) \to \Delta t$ in $\overrightarrow{G}^{\Delta t}$. Now, if the mapping $C : (v,u) \to c_{vu}$ is a constant for any edge $(v,u) \in E(G)$, $\overrightarrow{G}^C$ is called a *definitive constant flow* and abbreviated $\overrightarrow{G}^C$ to C.

(5) Integral

By definition, we know that

$$\frac{d}{dt} f\left(\overrightarrow{G}^L\right)[t] = F\left(\overrightarrow{G}^L[t]\right) \;\;\Rightarrow\;\; \frac{d}{dt} f\left(\overrightarrow{G}^L[t]+C\right) = F\left(\overrightarrow{G}^L[t]\right).$$

Thus, we generally define the *indefinite integral* of f on G-flow $\overrightarrow{G}^L[t]$ to be

$$\int F\left(\overrightarrow{G}^L[t]\right) dt \;=\; f\left(\overrightarrow{G}^L[t]\right) + C = \overrightarrow{G}^{f(L)+C}. \tag{3.19}$$

So, there is a relation of differential with the integral as follows:

$$\begin{aligned} \int \left(\frac{df}{dt}\left(\overrightarrow{G}^L[t]\right)\right) dt \;&=\; f\left(\overrightarrow{G}^L[t]\right) + C, \\ \frac{df}{dt}\left(\int \left(f\left(\overrightarrow{G}^L[t]\right)\right) dt\right) \;&=\; f\left(\overrightarrow{G}^L[t]\right). \end{aligned} \tag{3.20}$$

Define the definite integral of f by

$$\int_a^b f\left(\overrightarrow{G}^L[t]\right) dt \;=\; \lim_{\mu\to 0} \sum_{k=1}^{n} f\left(\overrightarrow{G}^L[\xi_i]\right) \cdot G^{\Delta t_k} \tag{3.21}$$

with

$$\int_a^b f\left(\overrightarrow{G}^L[t]\right) dt = F\left(\overrightarrow{G}^L[t]\right)\Big|_{t=b} - F\left(\overrightarrow{G}^L[t]\right)\Big|_{t=a}, \tag{3.22}$$

i.e., the fundamental theorem of calculus if

$$\frac{d}{dt}F\left(\overrightarrow{G}^{L}[t]\right) = f\left(\overrightarrow{G}^{L}[t]\right).$$

Notice that the evolving of G-flow is always keep G invariant, namely it is evolving with a formula

$$f\left(\overrightarrow{G}^{L}[\mathbf{x}]\right) \ = \ \overrightarrow{G}^{f(L)[\mathbf{x}]} \tag{3.23}$$

and get the differential and integral, i.e.,

$$\frac{df}{dt} \ = \ \lim_{\Delta t\to 0}\frac{f\left(\overrightarrow{G}^{L}[t+\Delta t]\right)-f\left(\overrightarrow{G}^{L}[t]\right)}{\overrightarrow{G}\Delta t} = \overrightarrow{G}^{\lim\limits_{\Delta t\to 0}\frac{f(L)[t+\Delta t]-f(L)[t]}{\Delta t}} = \overrightarrow{G}^{\frac{df(L)}{dt}}$$

and

$$\overrightarrow{G}^{\frac{df(L)}{dL}} \ = \ \overrightarrow{G}^{\lim\limits_{\Delta t\to 0}\frac{f(L)[t+\Delta t]-f(L)[t]}{\Delta t}} = \lim_{\Delta t\to 0}\frac{f\left(\overrightarrow{G}^{L}[t+\Delta t]\right)-f\left(\overrightarrow{G}^{L}[t]\right)}{\overrightarrow{G}\Delta t} = \frac{df}{dt}.$$

with more interesting formulas. For example, the exponential identity

$$e^{\overrightarrow{G}^{L}[\mathbf{x}]} = \mathbf{I} + \frac{\overrightarrow{G}^{L}[\mathbf{x}]}{1!} + \frac{\overrightarrow{G}^{2L}[\mathbf{x}]}{2!} + \cdots + \frac{\overrightarrow{G}^{nL}[\mathbf{x}]}{n!} + \cdots \tag{3.24}$$

on G-flow, which is a further generalization of identities

$$e^{x} \ = \ 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \tag{3.25}$$

and

$$e^{tA} \ = \ I_n + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots + \frac{A^n}{n!} + \cdots . \tag{3.26}$$

on a matrix A.

§4. Banach and Hilbert Space over a Graph Family

Notice that elements in the evolving of thing T are not always invariant but variant in a closed space. We should generalize G-flows to a more generalization, called *continuity flow* G^L with labeling in a Banach space, also with linear operators in case, called G-isomorphic operators, which have both the characters of graphs and vectors with metric, and can be applied to model the behavior of things T in the macroscopic and the microscopic, see [18]-[28], particularly [28] for details.

Definition 4.1([20],[28]) *A continuity flow* $\left(\overrightarrow{G}; L, \mathscr{A}\right)$ *is an oriented topological graph* $\overrightarrow{G}^{L}$ *in space* $\mathscr{S}$ *associated with a mapping* $L : v \to L(v)$, $(v,u) \to L(v,u)$, 2 *end-operators* $A^{+}_{vu} \in \mathscr{A} : L(v,u) \to L^{A^{+}_{vu}}(v,u)$ *and* $A^{+}_{uv} \in \mathscr{A} : L(u,v) \to L^{A^{+}_{uv}}(u,v)$ *on a Banach space* $\mathscr{B}$ *over a field*

$\mathscr{F}$ such as those shown in Figure 19 *with $L(v,u)=-L(u,v)$, $A^+_{vu}(-L(v,u))=-L^{A^+_{vu}}(v,u)$ for $\forall(v,u)\in E(\overrightarrow{G}^L)$.*

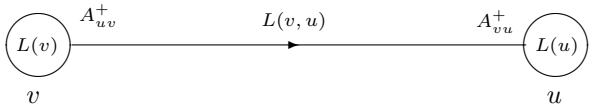

Figure 19

And meanwhile, holding with the continuity equation

$$\sum_{u\in N^-_G(v)} L^{A^+_{uv}}(u,v)-\sum_{u\in N^+_G(v)} L^{A^+_{uv}}(u,v) \;=\; L(v) \tag{4.1}$$

at any vertex $v\in V(\overrightarrow{G}^L)$ of topological graph $\overrightarrow{G}^L$, where $N^-_G(v),N^+_G(v)$ are respectively the in-neighborhood and out-neighborhood of vertex $v\in V(\overrightarrow{G}^L)$, namely all vertices in $N^-_G(v)\subset N_G(v)$ or $N^+_G(v)\subset N_G(v)$ flow into or out of the vertex v and $N^-_G(v)\cup N^+_G(v)=N_G(v)$.

Different from G-flow with an invariant G, by applying the operation of graph we consider $\overrightarrow{G}^L$ as a family of vectors underlying a topological graph $\overrightarrow{G}$ with addition, multiplication and scalar multiplication defined by

$$G^L+G'^{L'} \;=\; (G\setminus G')^L\bigcup\Big(G\bigcap G'\Big)^{L+L'}\bigcup(G'\setminus G)^{L'}, \tag{4.2}$$

$$G^L\cdot G'^{L'} \;=\; (G\setminus G')^L\bigcup\Big(G\bigcap G'\Big)^{L\cdot L'}\bigcup(G'\setminus G)^{L'}, \tag{4.3}$$

$$\lambda\cdot G^L \;=\; G^{\lambda\cdot L}. \tag{4.4}$$

where, for any vertex $v\in V(G)$ and edge $(v,u)\in E(G)$, $L(v),L'(v),L(v,u),L'(v,u)\in\mathscr{B}$, $L+L':v\to L(v)+L'(v)$, $(v,u)\to L(v,u)+L'(v,u)$, $L\cdot L':v\to L(v)\cdot L'(v)$, $(v,u)\to L(v,u)\cdot L'(v,u)$, $\lambda\cdot L:v\to\lambda\cdot L(v)$, $(v,u)\to\lambda\cdot L(v,u)$, $L(v)\cdot L'(v)$ and $L(v,u)\cdot L'(v,u)$ denotes the *Hadamard product* of vectors in Banach space $\mathscr{B}$, namely

$$(x_1,x_2,\cdots,x_n)\cdot(y_1,y_2,\cdots,y_n) \;=\; (x_1y_1,x_2y_2,\cdots,x_ny_n). \tag{4.5}$$

Let $\mathscr{G}$ be a graph family closed under the union operation of graph, $\mathscr{B}$ be a Banach space over field $\mathscr{F}$ and denoted by $\mathscr{G}_{\mathscr{B}}$ all continuity flows $\overrightarrow{G}^L$ with $\overrightarrow{G}\in\mathscr{G}$, $L:V(\overrightarrow{G})\bigcup E(\overrightarrow{G})\to\mathscr{B}$. Then, we have the result following.

Theorem 4.1([24],[25]) *If $\mathscr{G}$ is a closed family of graphs under the union operation and $\mathscr{B}$ a linear space $(\mathscr{B};+,\cdot)$, then, all continuity flows $(\mathscr{G}_{\mathscr{B}};+,\cdot)$ is a linear space, and furthermore, a commutative ring if $\mathscr{B}$ is a commutative ring $(\mathscr{B};+,\cdot)$ over a field $\mathscr{F}$.*

Now, assume all end-operators are continuous linear operators in $\mathscr{A}$ and define the norm

of a continuity flow $\overrightarrow{G}^L$ by

$$\left\|\overrightarrow{G}^L\right\| = \sum_{(v,u)\in E\left(\overrightarrow{G}\right)} \left\|L^{A^+_{vu}}(v,u)\right\|, \tag{4.6}$$

where $\|\cdot\|$ is the norm on Banach space $\mathscr{B}$. Then, we can verify the non-negative, homogeneity and the triangle inequality hold with $\mathscr{G}_{\mathscr{B}}$ and the non-negative, conjugacy and the linearity if $\mathscr{B}$ is further a Hilbert space, i.e.,

Theorem 4.2([24],[25]) *If $\mathscr{G}$ is a closed family of graphs under the union operation and $\mathscr{B}$ a Banach space $(\mathscr{B};+,\cdot)$, then, $\mathscr{G}_{\mathscr{B}}$ with linear operators A^+_{vu}, A^+_{uv} for $\forall(v,u)\in E\left(\bigcup\limits_{G\in\mathscr{G}}\overrightarrow{G}\right)$ is a Banach space, and furthermore, $\mathscr{G}_{\mathscr{B}}$ is a Hilbert space if $\mathscr{B}$ is a Hilbert space.*

Notice that the G-invariant property is the principle of mathematics over combinatorial structure (3.19), i.e.,

$$f\left(\overrightarrow{G}^L[\mathbf{x}]\right) \ = \ \overrightarrow{G}^{f(L[\mathbf{x}])},$$

But *how it performs on the graph family $\mathscr{G}$?* Answering this question need a convention on equivalent continuity flows following.

Convention 4.3([25]) *If $L(v,u)=\mathbf{0}$ for an edge $(v,u)\in E\left(\overrightarrow{G}^L\right)$, we always identify $\overrightarrow{G}^L$ with $\left(\overrightarrow{G}\backslash(v,u)\right)^L$, i.e., $\overrightarrow{G}^L=\left(\overrightarrow{G}\backslash(v,u)\right)^L$.*

By Convention 4.3, we can always choose the supgraph $\widehat{G}=\bigcup\limits_{G\in\mathscr{G}}G$ and the G-invariant property

$$f\left(G^L[\mathbf{x}]\right) \ = \ G^{f(L)[\mathbf{x}]}$$

to be $\widehat{G}$-invariant, i.e.,

$$f\left(\widehat{G}^L[\mathbf{x}]\right) \ = \ \widehat{G}^{f(L)[\mathbf{x}]} \tag{4.7}$$

and then, introduce the linear $\widehat{G}$-invariant operator on Banach flow space $\mathscr{G}_{\mathscr{B}}$. In this case, a G-isomorphic operator can be generally defined by

Definition 4.4([25],[28]) *A mapping $f: G_1^{L_1}\to G_2^{L_2}$ is a G-isomorphic operator between continuity flows $G_1^{L_1}$ and $G_2^{L_2}$ if*

(1) *there is an isomorphism $\varphi:\widehat{G}\to\widehat{G}$ with $\widehat{G}\supset G_1,G_2$ in graph;*

(2) *for $\forall(v,u)\in E(G_1)$ there is $L_2=f\circ\varphi\circ L_1$ but for $\forall(v,u)\in E(G_2\setminus G_1)$, $f:\mathbf{0}\to L_2(v,u)$ and for $\forall(v,u)\in E(G_1\backslash G_2)$ and $\forall(v,u)\in E(\widehat{G}\backslash(G_1\bigcup G_2))$, $f:L(v,u)\to\mathbf{0}$*

and a G-isomorphic operator $f:\mathscr{G}_{\mathscr{B}}\to\mathscr{G}_{\mathscr{B}'}$ is linear if

$$f\left(\lambda\overrightarrow{G}_1^{L_1}+\mu\overrightarrow{G}_2^{L_2}\right) \ = \ f\left(\lambda\overrightarrow{G}_1^{L_1}\right)+f\left(\mu\overrightarrow{G}_2^{L_2}\right), \tag{4.8}$$

is continuous if

$$\|\overrightarrow{G}_1^{L_1} - \overrightarrow{G}_0^{L_0}\| < \delta(\varepsilon) \ \Rightarrow \ \|f\left(\overrightarrow{G}_1^{L_1}\right) - f\left(\overrightarrow{G}_0^{L_0}\right)\| < \varepsilon, \tag{4.9}$$

is bounded if there exists a constant $\xi \in [0, +\infty)$ *such that*

$$\|f\left(\overrightarrow{G}_1^{L_1}\right) - f\left(\overrightarrow{G}_2^{L_2}\right)\| < \xi\|\overrightarrow{G}_1^{L_1} - \overrightarrow{G}_2^{L_2}\|. \tag{4.10}$$

By definition, $\mathscr{G}_{\mathscr{B}}^{\pm}$ is all harmonic flows G^L with $\overrightarrow{G} \in \mathscr{G}$, $L : v \to (L(v), -L(v))$ for $v \in E(\overrightarrow{G})$ and, $L : (v,u) \to (L(v,u), -L(v,u))$, the action of end-operators $A_{vu}^{+} : (L(v,u), -L(v,u)) \to (L^{A_{vu}^{+}}(v,u), -L^{A_{vu}^{+}}(v,u))$, $A_{uv}^{+} : (L(v,u), -L(v,u)) \to (L^{A_{uv}^{+}}(v,u), -L^{A_{uv}^{+}}(v,u))$. Now, if we do not label $\overrightarrow{G}$ with 2-pairs $(L(v), -L(v)), (L(v,u), -L(v,u))$ but only $L(v), L(v,u)$ for $v \in V(G)$ and $(v,u) \in E(G)$, then $\mathscr{G}_{\mathscr{B}}^{\pm}$ is nothing else but continuity flow space $\mathscr{G}_{\mathscr{B}}$, and a linear operator $\mathbf{T} : \mathscr{G}_{\mathscr{B}}^{\pm} \to \mathscr{G}_{\mathscr{B}}^{\pm}$ to be $\mathbf{T} : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$ with $\widehat{G}$ invariant, i.e., a G-isomorphic operator on on continuity flow space $\mathscr{G}_{\mathscr{B}}$, and all proofs in [22] hold on $\mathscr{G}_{\mathscr{B}}$ also.

Theorem 4.5(Fixed Flow Theorem) *For a continuous G-isomorphic contractor $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}'}$ there is only one continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$ such that $f\left(G^L\right) = G^L$.*

Proof It is inferred from the proof of Theorem 2.8 on harmonic flows in [22]. □

Theorem 4.6(Banach Inverse Theorem) *A G-isomorphic linear operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$ is continuous if and only if it is bounded and furthermore, if f is $1-1$ then the inverse operator f^{-1} of f is also a G-isomorphic continuous operator.*

Proof It is inferred from the proof of Theorem 2.10 on harmonic flows in [22]. □

For a G-isomorphic operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$, its image Grap$f$ of is defined by

$$\text{Grap}f \ = \ \left\{ \left(\overrightarrow{G}^L, f\left(\overrightarrow{G}^L\right)\right) \middle| \overrightarrow{G}^L \in \mathscr{G}_{\mathscr{B}} \right\} \tag{4.11}$$

and f is *closed* if the image Grap$f$ of f is closed.

Theorem 4.7(Closed Graph Theorem) *If $\mathbf{T} : \mathscr{G}_{\mathscr{B}_1} \to \mathscr{G}_{\mathscr{B}_2}$ is a closed linear operator with Banach spaces $\mathscr{B}_1$, $\mathscr{B}_2$, then $\mathbf{T}$ is continuous.*

Proof It is inferred from the proof of Theorem 2.13 on harmonic flows in [22]. □

Particularly, a G-isomorphic linear operator $f : \mathscr{G}_{\mathscr{B}} \to \mathbb{R}$ or $\mathbb{C}$ is called a *flow functional*, which can be applied to generalize the Hahn-Banach theorem to Banach flow space $\mathscr{G}_{\mathscr{B}}$.

Theorem 4.8(Hahn-Banach Theorem) *Let $\mathscr{H}_{\mathscr{B}}$ be a subspace of Banach flow space $\mathscr{G}_{\mathscr{B}}$ and let $F : \mathscr{H}_{\mathscr{B}} \to \mathbb{C}$ be a continuous linear flow functional on $\mathscr{H}_{\mathscr{B}}$. Then, there is a continuous linear flow functional $\widetilde{F} : \mathscr{G}_{\mathscr{B}} \to \mathbb{C}$ satisfies the conditions that if $\overrightarrow{G}^L \in \mathscr{H}_{\mathscr{B}}$ then $\widetilde{F}(\overrightarrow{G}^L) = F(\overrightarrow{G}^L)$ and $\left\|\widetilde{F}\right\| = \|F\|$.* Particularly, *if $\mathbf{O} \neq \overrightarrow{G}_0^{L_0} \in \mathscr{G}_{\mathscr{B}}$, there is a continuous linear flow functional*

F such that $\|F\| = 1$ and $\|F(\overrightarrow{G}_0^{L_0})\| = \|\overrightarrow{G}_0^{L_0}\|$.

Proof It is inferred from the proof of Theorem 2.23 on harmonic flows in [22]. □

Particularly, let $\mathscr{G}_{\mathscr{B}}$ be a Hilbert flow space. Then, we know

Theorem 4.9(Fréchet-Riesz Theorem]) *Let $f : \mathscr{G}_{\mathscr{B}} \to \mathbb{C}$ be a continuous linear flow functional. For any continuous flow $\overrightarrow{G}^L \in \mathscr{G}_{\mathscr{B}}$, there uniquely exists a continuous flow of $\overrightarrow{G'}^{L'} \in \mathscr{G}_{\mathscr{B}}$ holding with*

$$f\left(\overrightarrow{G}^L\right) = \left\langle \overrightarrow{G}^L, \overrightarrow{G'}^{L'} \right\rangle. \tag{4.12}$$

Proof It is proved on G-flows by the proof of Theorem 3.5 in [16], which can be naturally generalized to continuity flow similarly. Firstly, let

$$\mathscr{G}_{\mathscr{B}} \ = \ \mathcal{N}(f) + \mathcal{N}^+(f)$$

be an orthogonal decomposition of $\mathscr{G}_{\mathscr{B}}$ and $\overrightarrow{G}^{L'} = \lambda \overrightarrow{G}^L$ with $\lambda = \overline{f(\overrightarrow{G}^L)}/\|\overrightarrow{G}^L\|$ such that

$$f\left(\overrightarrow{G}^L\right) = \left\langle \overrightarrow{G}^L, \overrightarrow{G'}^{L'} \right\rangle$$

and if there is another $\overrightarrow{G}^* \in \mathscr{G}_{\mathscr{B}}$ holding with (4.10), then there must be

$$\begin{aligned} \left\langle \overrightarrow{G}^L, \overrightarrow{G'}^{L'} \right\rangle \ &= \ \left\langle \overrightarrow{G}^L, \overrightarrow{G*}^{L*} \right\rangle \\ &\Rightarrow \ \left\langle \overrightarrow{G}^L, \overrightarrow{G'}^{L'} - \overrightarrow{G*}^{L*} \right\rangle \ = \ \mathbf{O} \end{aligned}$$

i.e., $\overrightarrow{G'}^{L'} = \overrightarrow{G*}^{L*}$, the uniqueness of $\overrightarrow{G'}^{L'}$. □

Furthermore, we can establish differential theory on Hilbert flow space $\mathscr{G}_{\mathscr{B}}$ and obtain the dynamic Euler-Lagrange equations on a thing T, i.e., let $\overrightarrow{G}^L \in \mathscr{G}_{\mathscr{B}}$ with $L : (v,u) \to L(v,u)$ for $(v,u) \in E(G^L)$ and all end-operators in $\mathscr{A}$ satisfying $[A, \frac{\partial}{\partial x_i}] = \mathbf{0}$ for $\forall A \in \mathscr{A}$. In this case, we can also define differential and integral operators by

$$\partial_i : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}} \quad \text{and} \quad \int_C \overrightarrow{G}^{L^2} dz = \overrightarrow{G}^{\int_C L^2 dz} \tag{4.13}$$

for integers $1 \leq i \leq n$. Certainly, they are all $\widehat{G}$-isomorphic operators.

Now, let $\mathscr{L}\left[L(t, \mathbf{x}(t), \dot{\mathbf{x}}(t))\right] : (v,u) \in E(G) \to \mathscr{L}\left[L(t, \mathbf{x}(t), \dot{\mathbf{x}}(t))\right](v,u)$ be a differentiable functional with $[\mathscr{L}, A] = \mathbf{0}$ for all end-operators $A \in \mathscr{A}$. Then, we know that

$$\frac{\partial \overrightarrow{G}^{\mathscr{L}}}{\partial x_i} - \frac{d}{dt}\frac{\partial \overrightarrow{G}^{\mathscr{L}}}{\partial \dot{x}_i} = \mathbf{O}, \qquad 1 \leq i \leq n \tag{4.14}$$

by the proof of Theorem 3.4 on harmonic flows L^2 in [22]. Particularly, if $\mathscr{L}\left[\overrightarrow{G}^L[t]\right]$ of continuity flow $\overrightarrow{G}^L[t]$ is independent on (v,u), i.e., they are synchronized then the dynamic behavior of $\overrightarrow{G}^L[t]$ can be characterized by n equations

$$\frac{\partial L}{\partial x_i}-\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}=0,\qquad 1\leq i\leq n, \tag{4.15}$$

i.e., the Euler-Lagrange equations, which establishes graph dynamics suggested in [2].

So, *what is the importance of Frechet-Riesz theorem in Hilbert flow space?* Certainly, the subdividing on thing T can be kept going with the technological progress. For example, the models on proton, neutron and meson by quarks are shown in Figure 20.

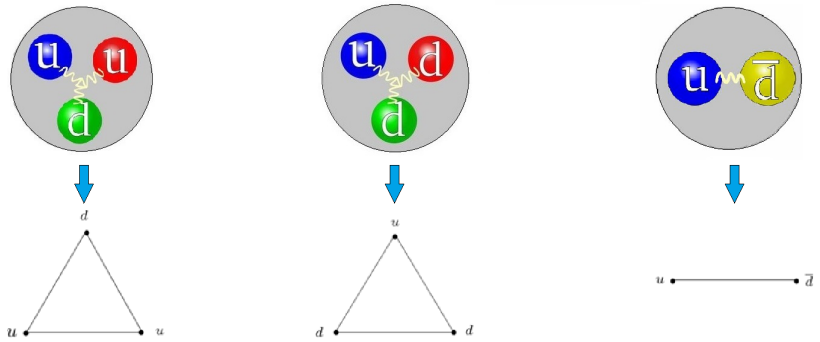

Figure 20

Now, *could we conclude the* 3 *hypothesis in quantum mechanics inherited with combinatorial structure are also true?* The answer is certainly Yes because the Frechet-Riesz theorem in Hilbert flow space $\mathscr{G}_{\mathscr{B}}$ concludes that the existence of Hermitian operator H in a quantum Q with quark structure $\overrightarrow{G}^L$, i.e., the correctness of the 3 hypothesis in quantum mechanics with combinatorial structure in reductionism.

§5. Conclusion

There are 2 motivations for the development of mathematics, i.e., one is the need of human recognition on the universe and another is solving problems within mathematics itself such as the Hilbert's 23 problems. In fact, most mathematicians spend their whole life on the second but few one on the first, i.e., the original intention of mathematics, which resulted in Einstein's complaint words on mathematics, i.e.,

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

Certainly, Einstein's complaint on mathematics implies that the developing of mathematics can not only rely on solving problems in mathematics itself, which is only the mathematical perfection and rather than the game pastime of our mathematicians but also the return to the original needs of recognition of human because mathematics is providing the recognitive way of human on the nature and promoting the sustainable development for human, i.e., the harmony

of human with the nature, including the correctly understanding the nature and following the nature. Among the recognitive ways, the reductionism is a systemic mechanism. However, we lack mathematics to reductionism in the classic. In fact, the harmony of human with the nature needs the human to refrain from disturbing nature, which results in the continuity flow or simply $\overrightarrow{G}$-flow $\overrightarrow{G}^L$ in human activities, i.e., substance flow with conservative laws of nature in microscopic ([27]), corresponding to the reductionism and the establishing of technological systems for cyclical use of non-renewable resources ([29]) in order to achieve the conservation of substance flow in local region or the global.

References

[1] G.Birkhoff and S.MacLane, *A Survey of Modern Algebra* (4th edition), Macmillan Publishing Co., Inc, 1977.

[2] G.R.Chen, X.F.Wang and X.Li, *Introduction to Complex Networks – Models, Structures and Dynamics* (2nd Edition), Higher Education Press, Beijing, 2015.

[3] John B.Conway, *A Course in Functional Analysis*, Springer-Verlag New York, Inc., 1990.

[4] J.L.Gross and T.W.Tucker, *Topological Graph Theory*, John Wiley & Sons, 1987.

[5] Yanpei Liu,*Enumerative Theory of Maps*, Kluwer Academic Publisher, Dordrecht /Boston/ London (1999).

[6] Linfan Mao, *Automorphism Groups of Maps, Surfaces and Smarandache Geometries*, First edition published by American Research Press in 2005, Second edition is a graduate textbook in mathematics, published by The Education Publisher Inc., USA, 2011.

[7] Linfan Mao, *Smarandache Multi-Space Theory*, First edition published by Hexis, Phoenix in 2006, Second edition is a graduate textbook in mathematics, published by The Education Publisher Inc., USA, 2011.

[8] Linfan Mao, Combinatorial speculation and combinatorial conjecture for mathematics, *International J.Math. Combin.* Vol.1(2007), No.1, 1-19.

[9] Linfan Mao, Geometrical theory on combinatorial manifolds, *JP J.Geometry and Topology*, Vol.7, No.1(2007),65-114.

[10] Linfan Mao, *Combinatorial Geometry with Applications to Field Theory*, First edition published by InfoQuest in 2009, Second edition is a graduate textbook in mathematics, published by The Education Publisher Inc., USA, 2011.

[11] Linfan Mao, Combinatorial fields – An introduction, *International J. Math.Combin.*, Vol.3 (2009), 1-22.

[12] Linfan Mao, Graph structure of manifolds with listing, *International J.Contemp.Math. Science*, Vol.5, No.2(2011), 71-85.

[13] Linfan Mao, Global stability of non-solvable ordinary differential equations with applications, *International J. Math.Combin.*, Vol.1, 2013, 1-37.

[14] Linfan Mao, Mathematics on non-mathematics – A combinatorial contribution, *International J.Math. Combin.*, Vol.3(2014), 1-34.

[15] Linfan Mao, A review on natural reality with physical equations, *Progress in Physics*, Vol.11, 3 (2015),276-282.

[16] Linfan Mao, Extended Banach G-flow spaces on differential equations with applications, *Electronic J.Mathematical Analysis and Applications*, Vol.3, No.2(2015), 59-91.

[17] Linfan Mao, Cauchy problem on non-solvable system of first order partial differential equations with applications, *Methods and Applications of Analysis*, Vol. 22, 2（2015）, 171 – 200.

[18] Linfan Mao, Mathematics with natural reality – Action flows, *Bull.Cal.Math.Soc.*, Vol.107, 6(2015), 443-474.

[19] Linfan Mao, Biological n-system with global stability, *Bull.Cal.Math.Soc.*, Vol.108, 6(2016), 403-430.

[20] Linfan Mao, Complex system with flows and synchronization, *Bull.Cal.Math.Soc.*, Vol.109, 6(2017), 461 – 484.

[21] Linfan Mao, *Mathematical Reality – My Philosophy on Mathematics with Reality*, The Education Publisher Inc., USA, 2018.

[22] Linfan Mao, Harmonic flow's dynamics on animals in microscopic level with balance recovery, *International J.Math. Combin.*, Vol.1(2019), 1-44.

[23] Linfan Mao, Graphs, networks and natural reality – From intuitive abstracting to theory, *International J.Math. Combin.*, Vol.4(2019), 1-18.

[24] Linfan Mao, Mathematical elements on natural reality, *Bull.Cal.Math.Soc.*, Vol.111, 6(2019), 597 – 618.

[25] Linfan Mao, Dynamic network with e-index applications，*International J.Math.Combin.*, Vol.4 (2020), 1-35.

[26] Linfan Mao, Reality with Smarandachely denied axiom, *International J.Math. Combin.*, Vol.3(2021), 1-19.

[27] Linfan Mao, Combinatorial science – - How science leads humans with the nature in harmony, *International J.Math. Combin.*, Vol.3(2023), 1-15.

[28] Linfan Mao, *Combinatorial Theory on the Universe*, Global Knowledge-Publishing House, 2023.

[29] Linfan Mao, *Field Theory on the Universe – Field Action in Emperor' s Inner Canon* (in Chinese), Global Knowledge-Publishing House, 2025.

[30] William S.Massey, *Algebraic Topology: An Introduction*, Seringer-Verlag, New York, 1967.

[31] B.Mohar and C.Thomassen, *Graphs on Surfaces*, The Johns Hopkins University Press, London, 2001.

[32] Quang Ho-Kim and Pham Xuan Yem, *Elementary Particles and Their Interactions: Conceptions and Phenomena*, Springer-Verlag Berlin Heidelberg, 1998.

[33] F.Smarandache, *A Unifying Field in Logics–Neutrosopy: Neturosophic Probability, Set, and Logic*, American research Press, Rehoboth, 1999.

[34] F.Smarandache, Mixed non-Euclidean geometries, *eprint arXiv: math/0010119*, 10/2000.

[35] F.Smarandache, S-denying a theory, *International J.Math. Combin.*, Vol.2(2013), 1-7.

[36] W.B.Vasantha Kandasamy, *Bialgebraic Structures and Smarandache Bialgebraic Structures*, American Research Press, 2003.

[37] W.B.Vasantha Kandasamy and F.Smarandache, *Basic Neutrosophic Algebraic Structures and Their Applications to Fuzzy and Neutrosophic Models*, Hexis, Church Rock, 2004.

[38] W.B.Vasantha Kandasamy and F.Smarandache, *N-Algebraic Structures and S-N-Algebraic Structures*, HEXIS, Phoenix, Arizona, 2005.

[39] Q.Yang, *Animal Histology and Embryology*(2nd Edition), China Agricultural University Press, 2018.

[40] Z.C.Zhang, *Comments on Emperor's Inner Canon*(Qing Dynasty, in Chinese), Northern Literature and Art Publishing House, 2007.